Collision and Contact

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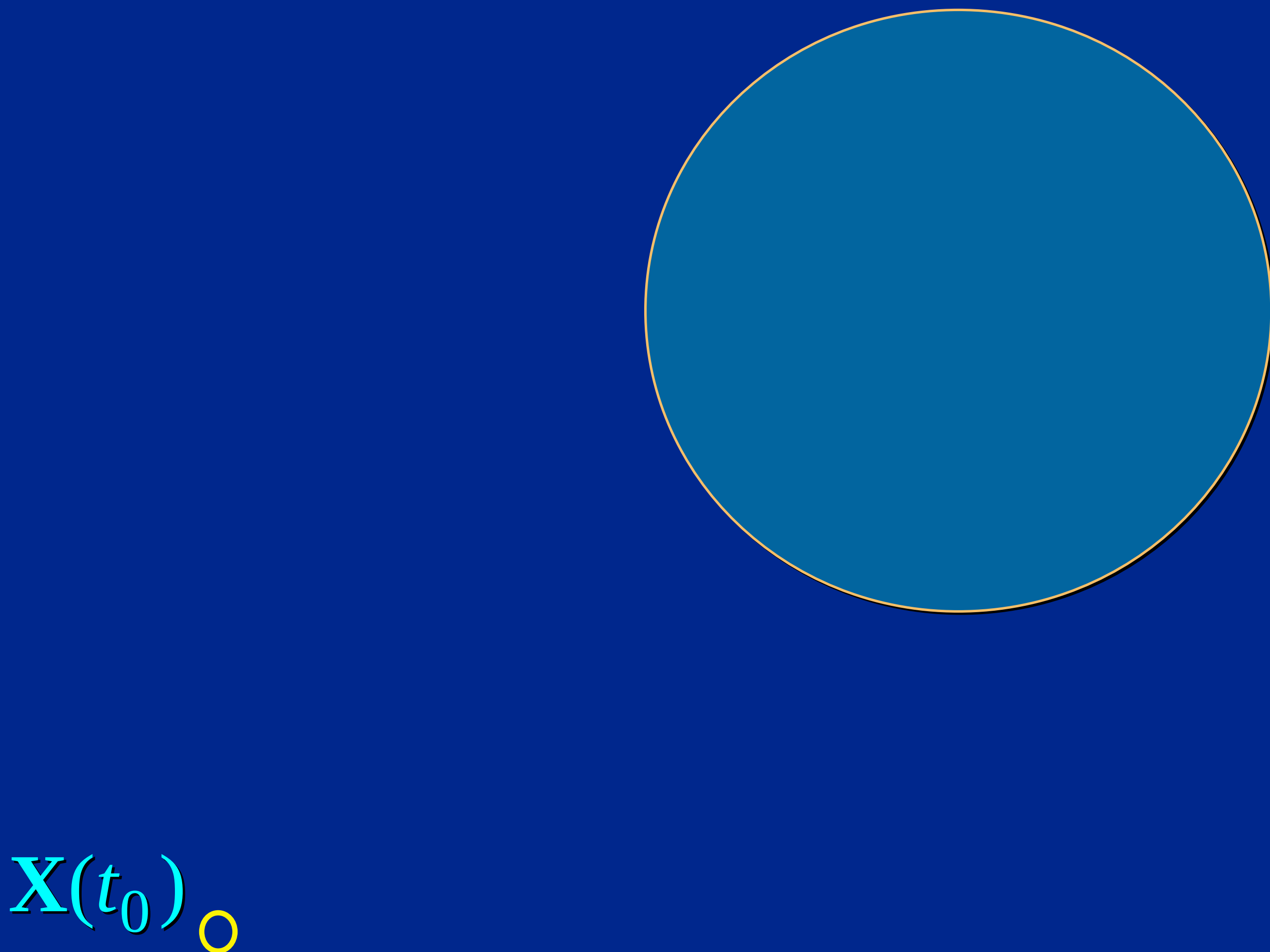
PIXAR ANIMATION STUDIOS

Collision and Contact

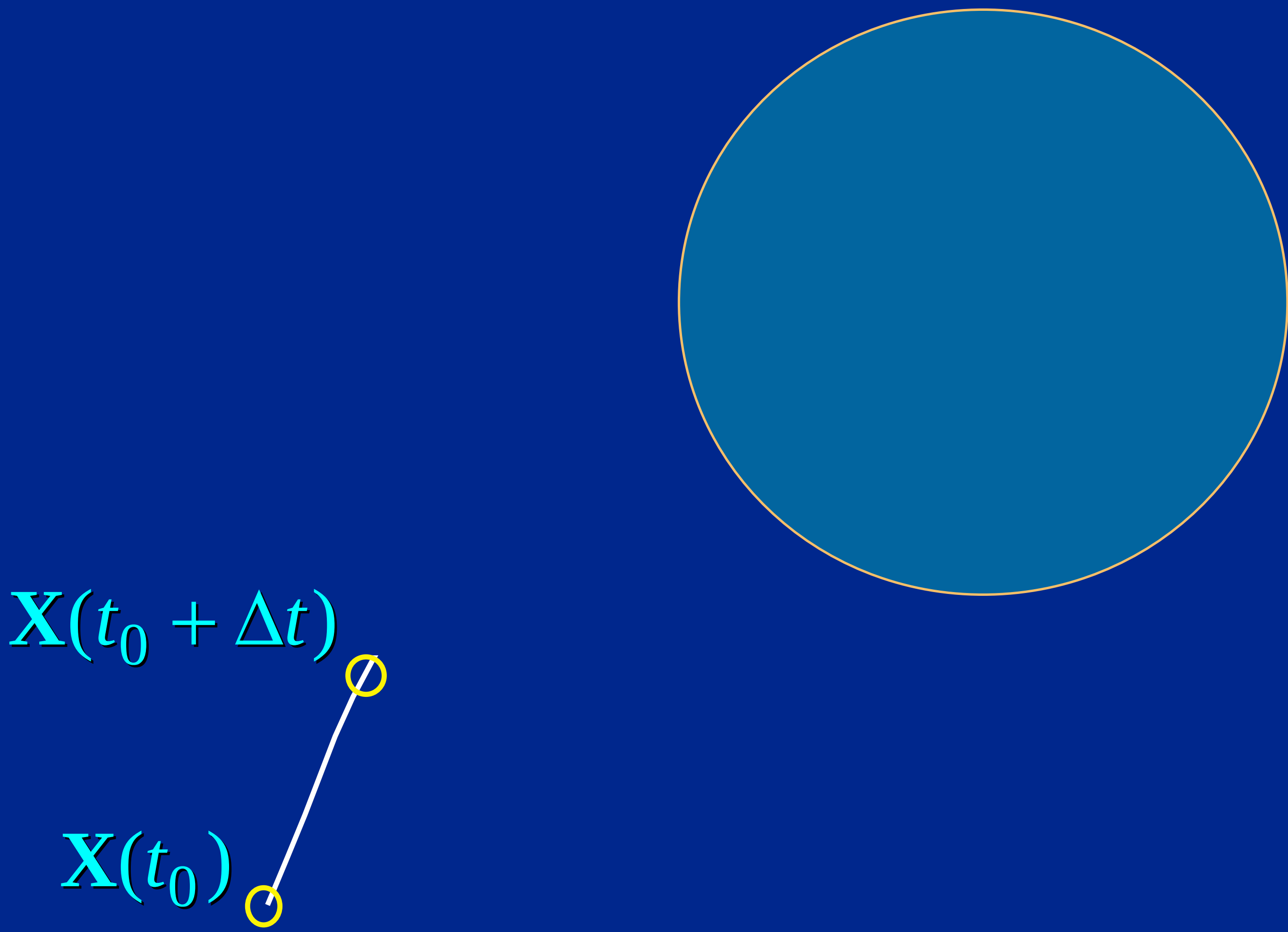
We want objects to behave as if they were solid and not interpenetrate. When collisions or contacts occur we need to:

- Detect them.
- Fix them (if they're wrong).
- Maintain them.

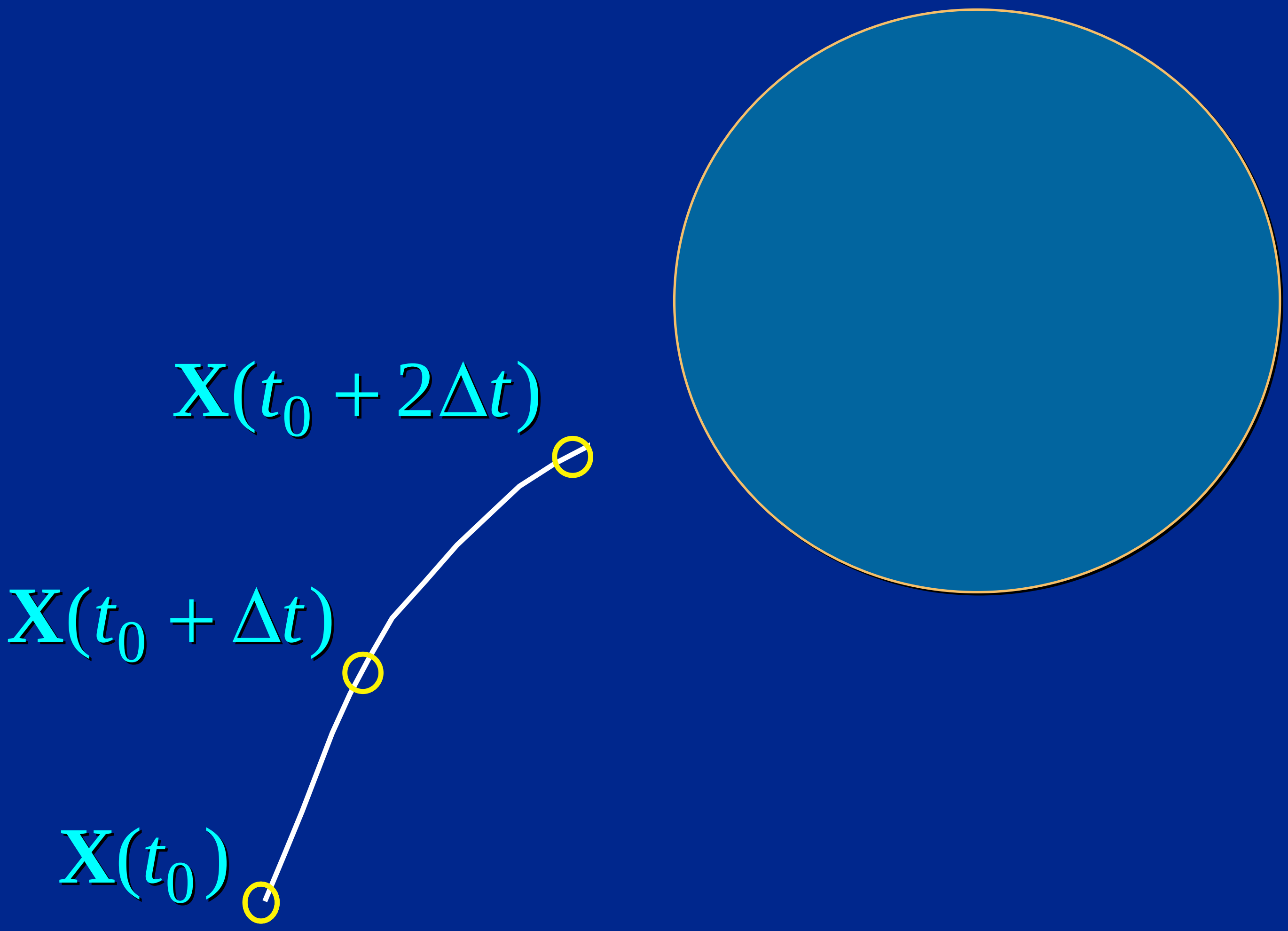
Simulations with Collisions



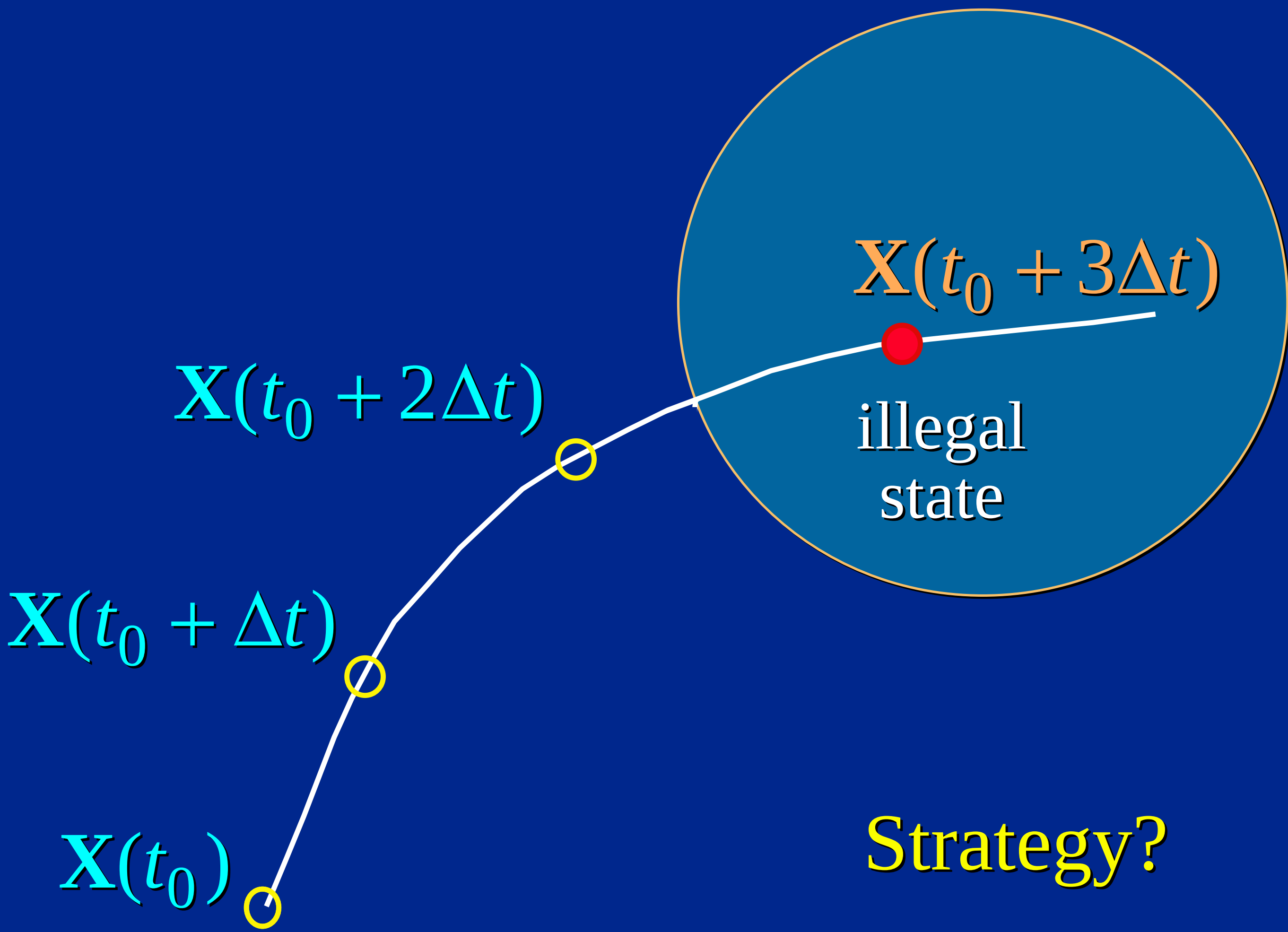
Simulations with Collisions



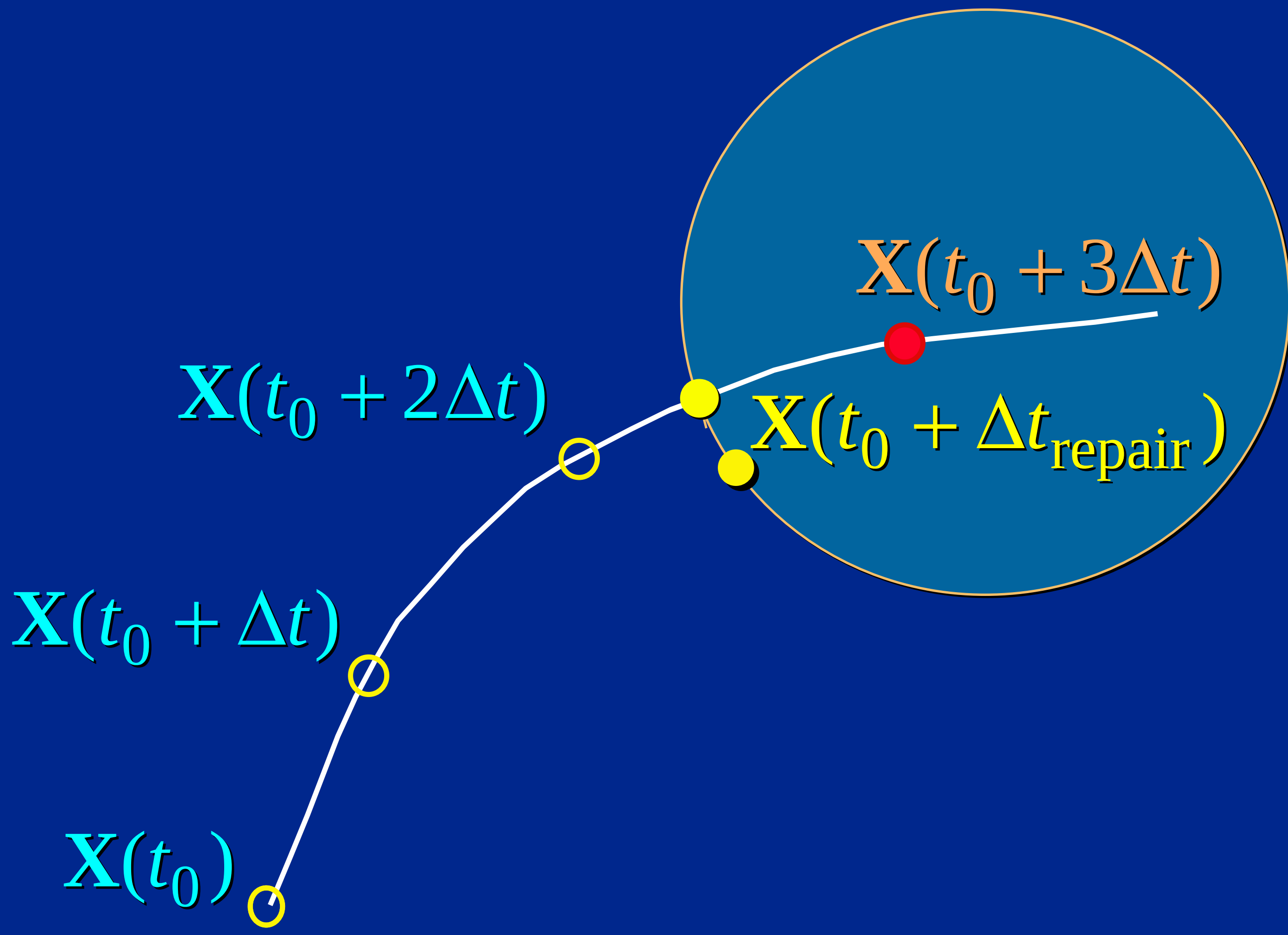
Simulations with Collisions



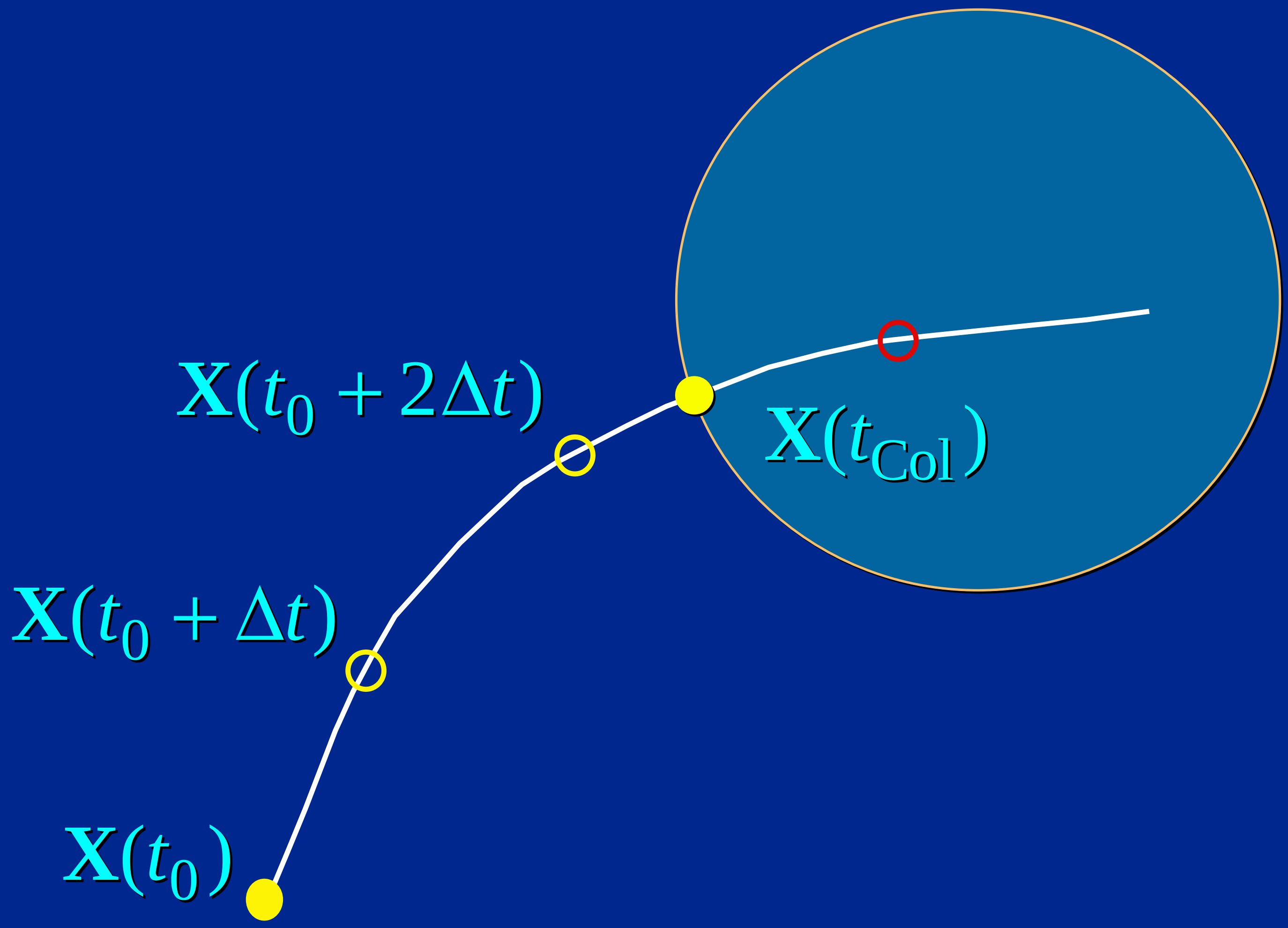
An Illegal State X



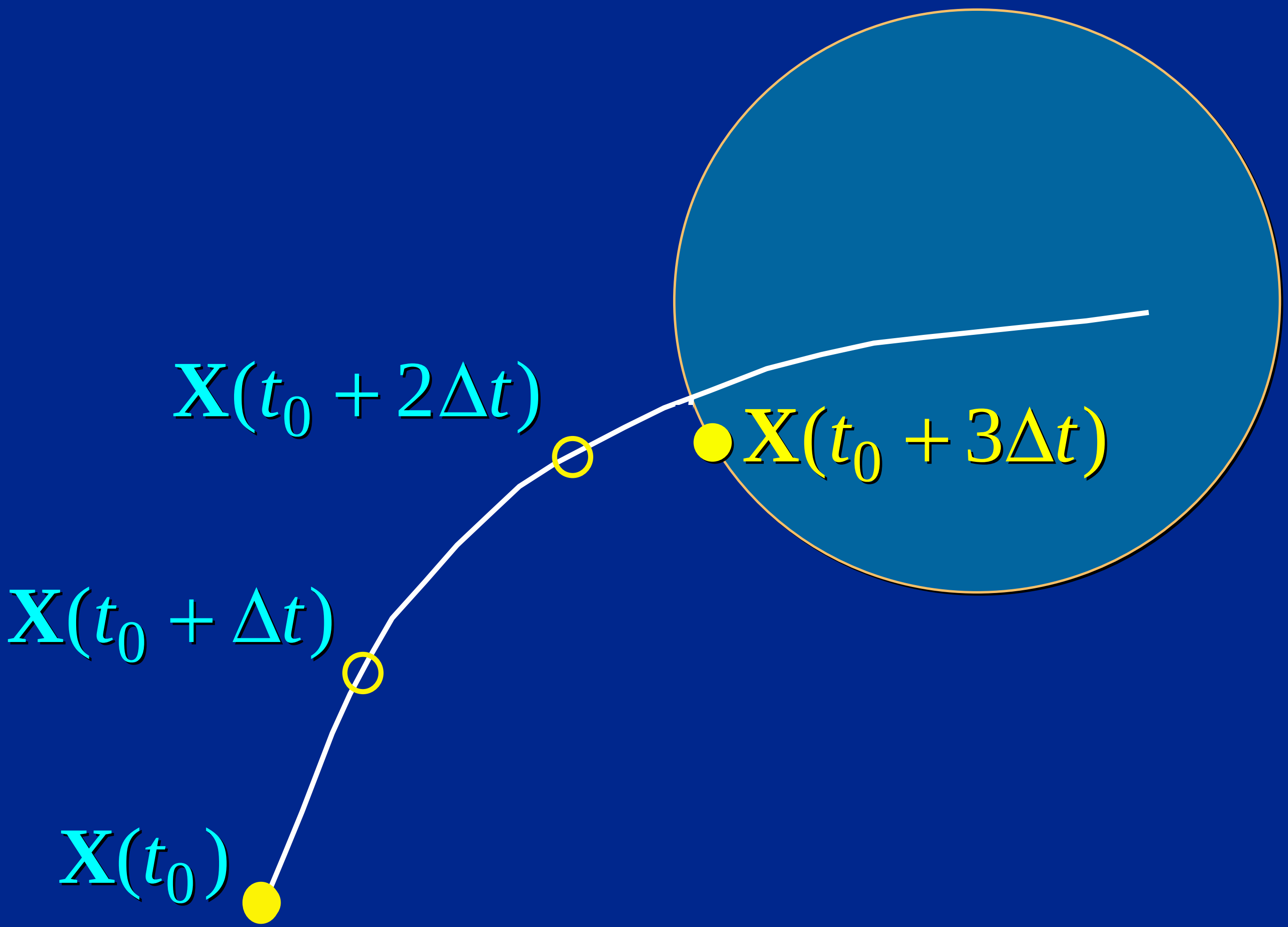
Plan 1: Gradual Repair



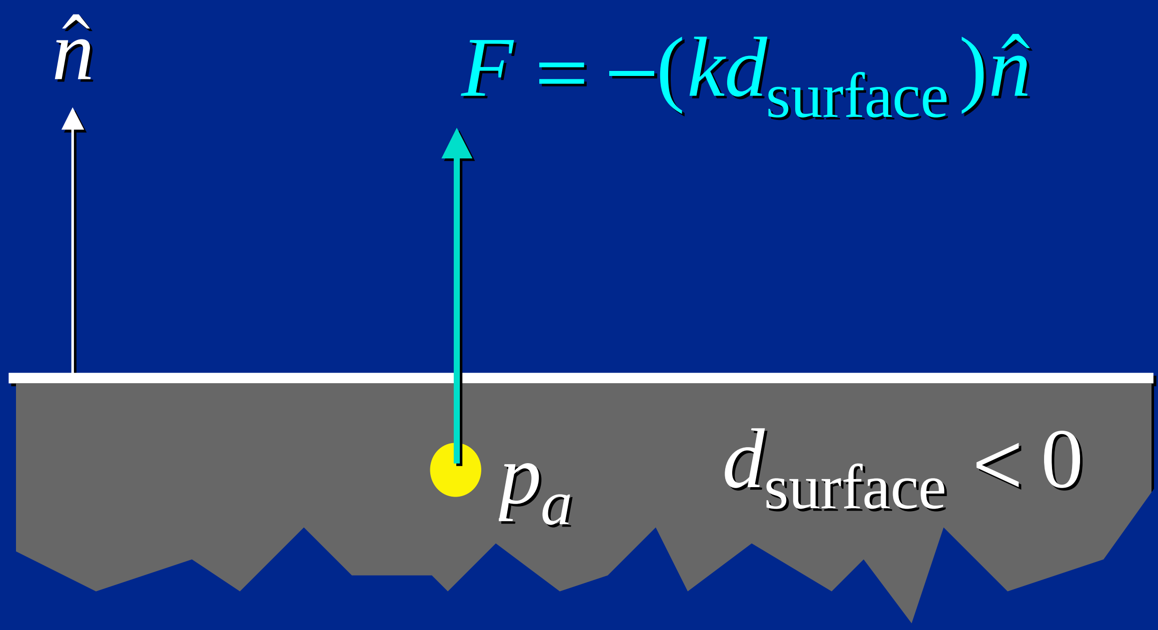
Plan 2: Backstep



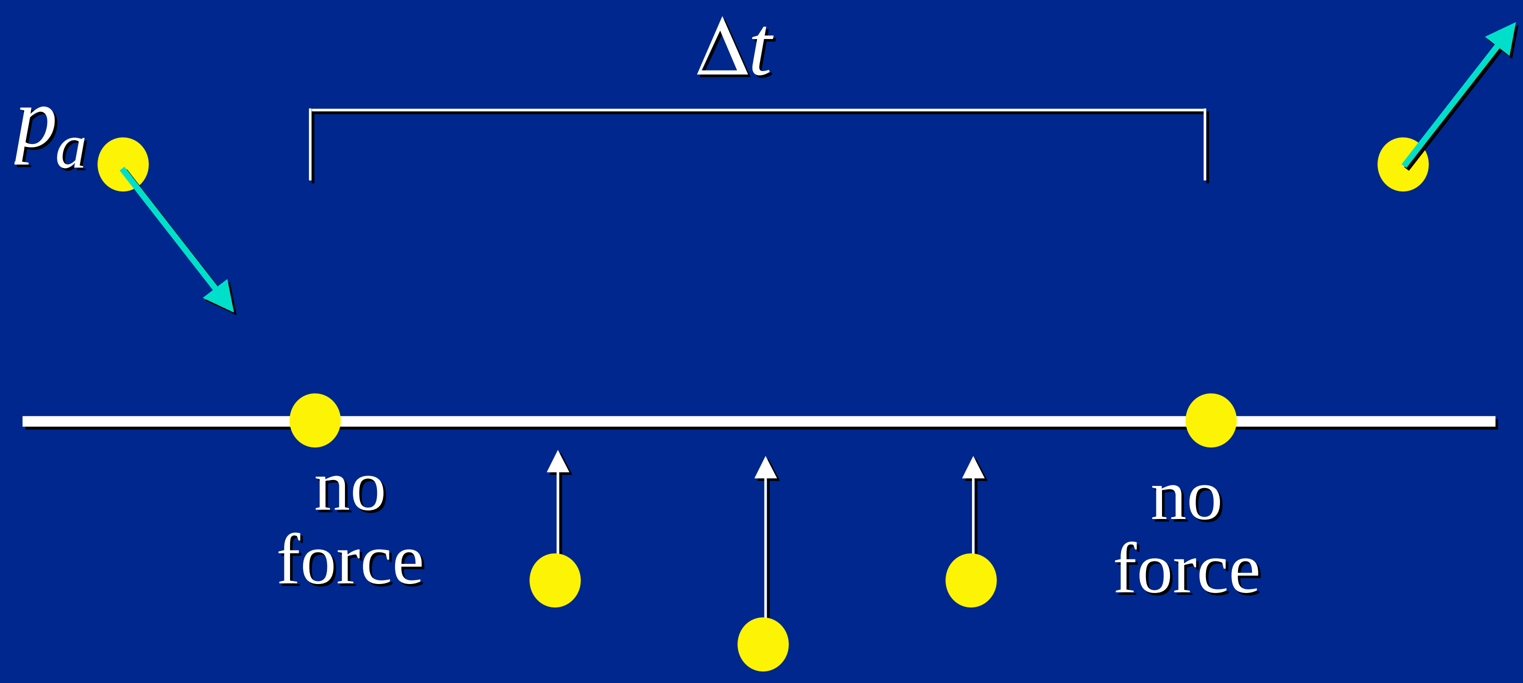
Plan 3: Just Lie About It!



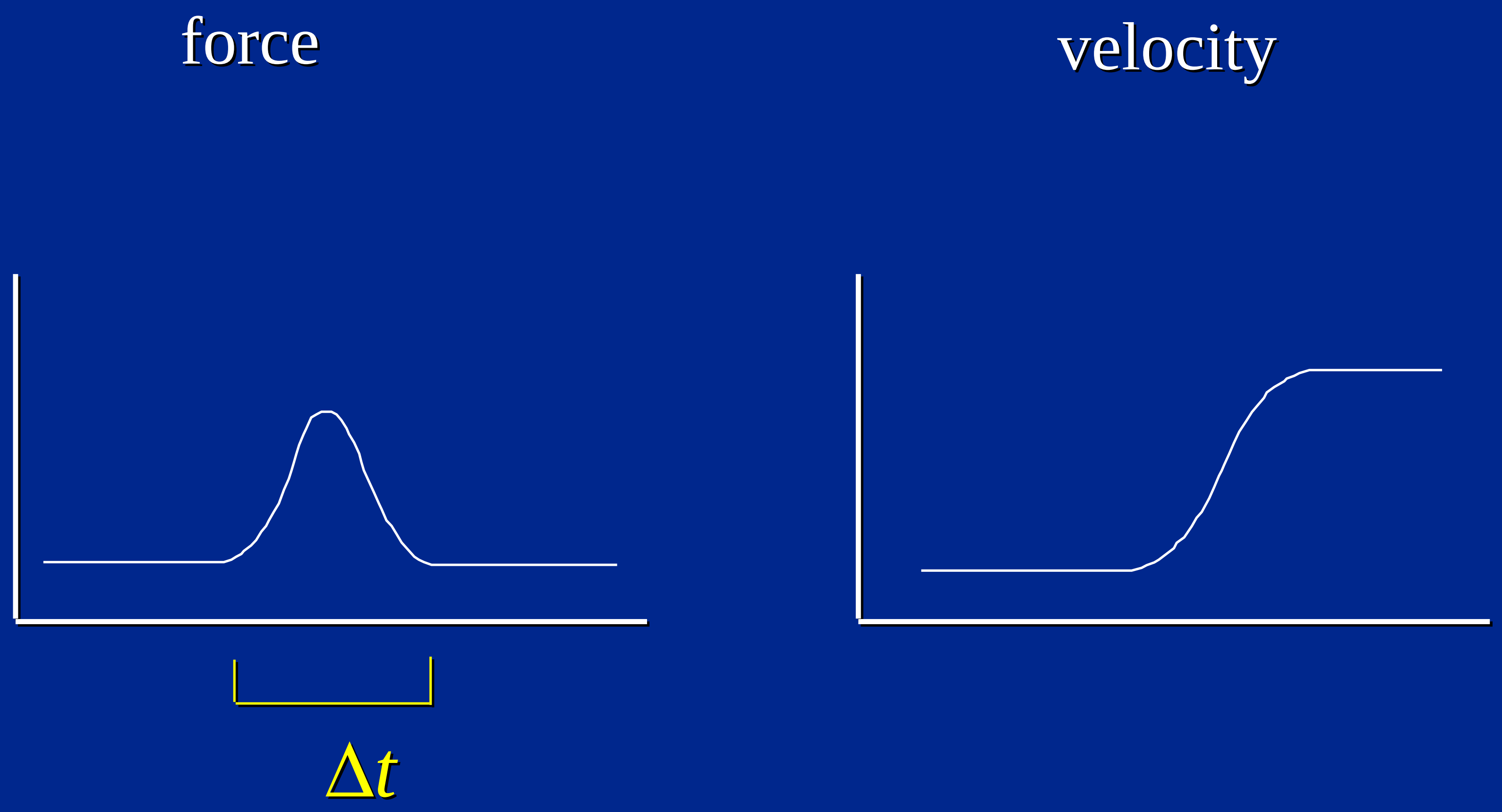
Penalty-Method Approach



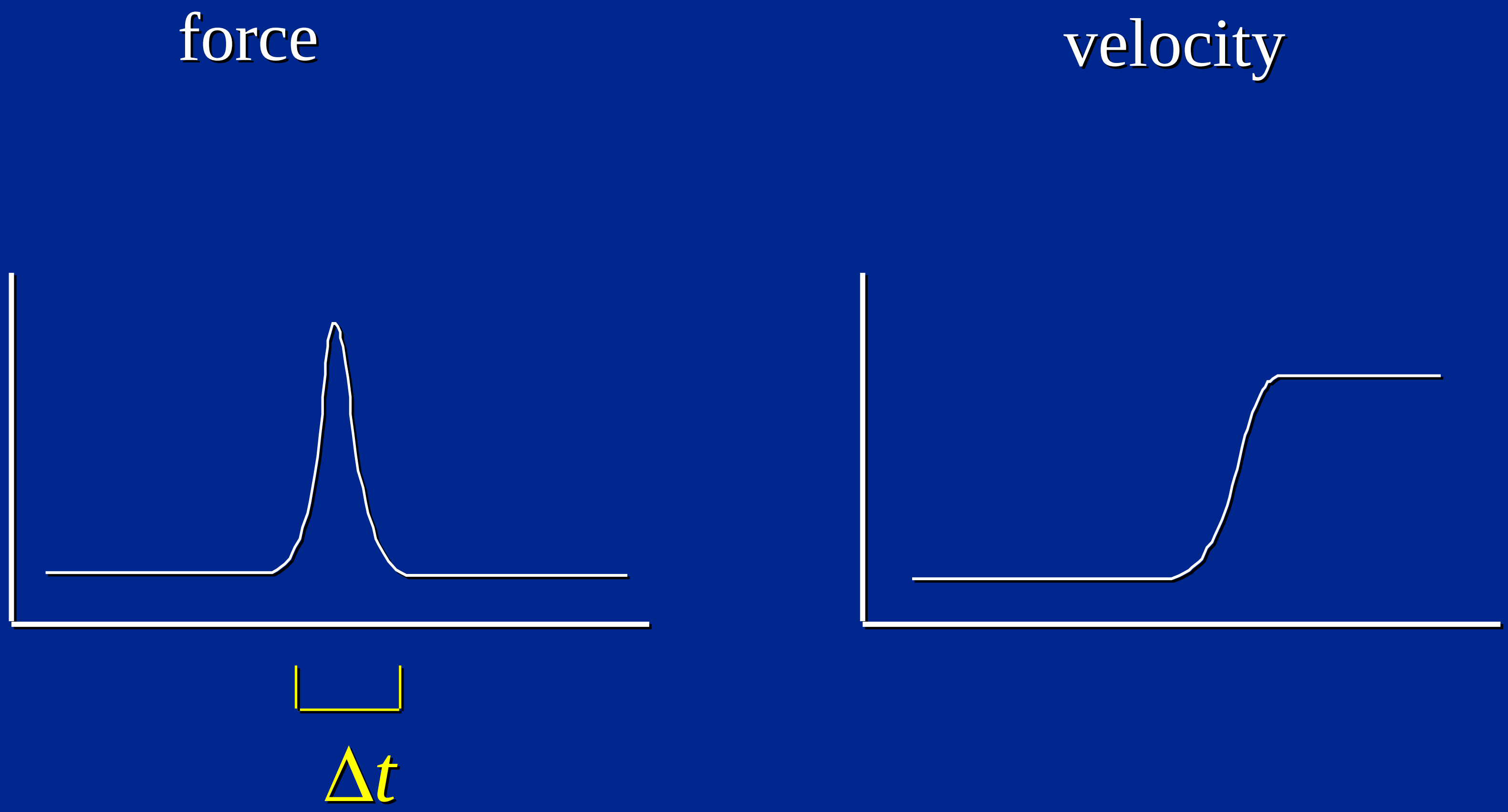
Collision Process



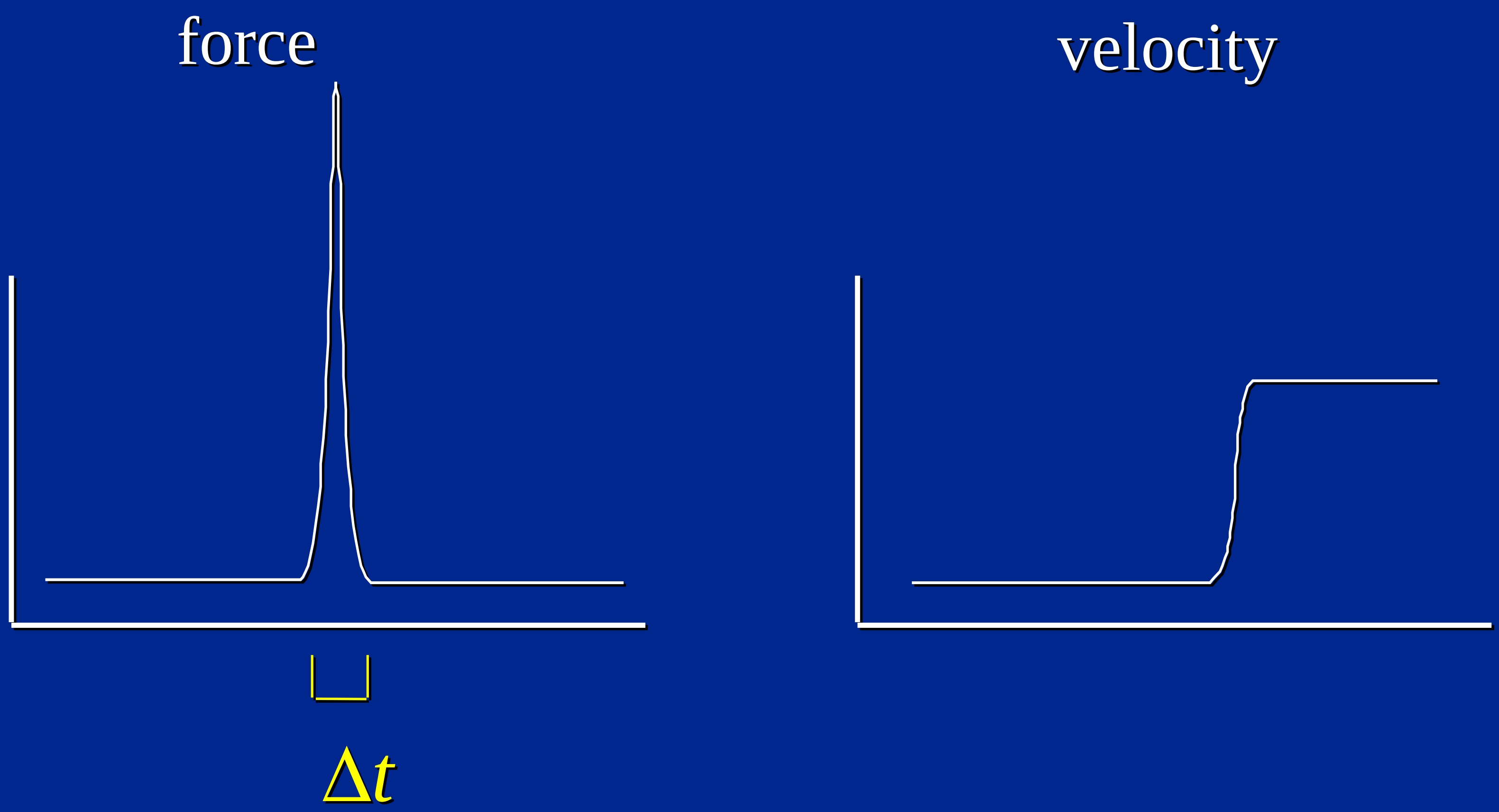
A Soft Collision



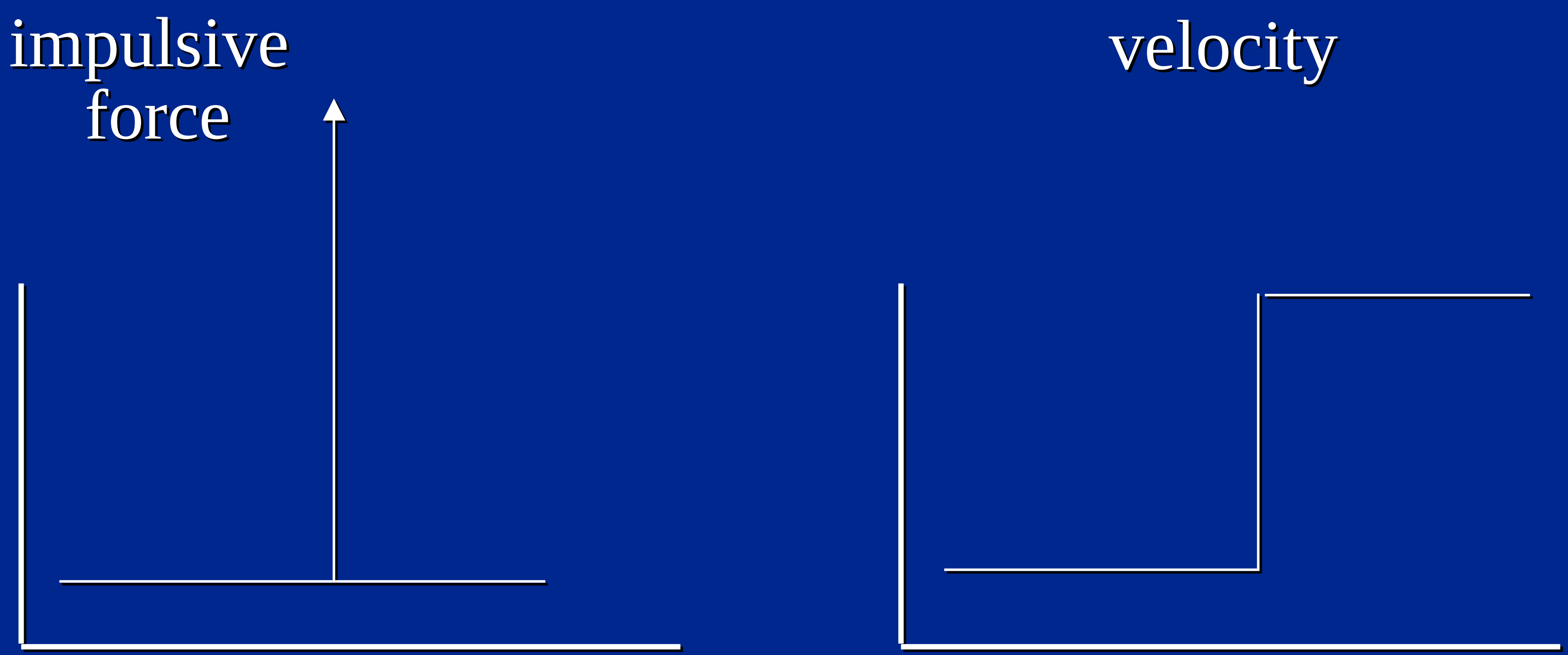
A Harder Collision



A Very Hard Collision



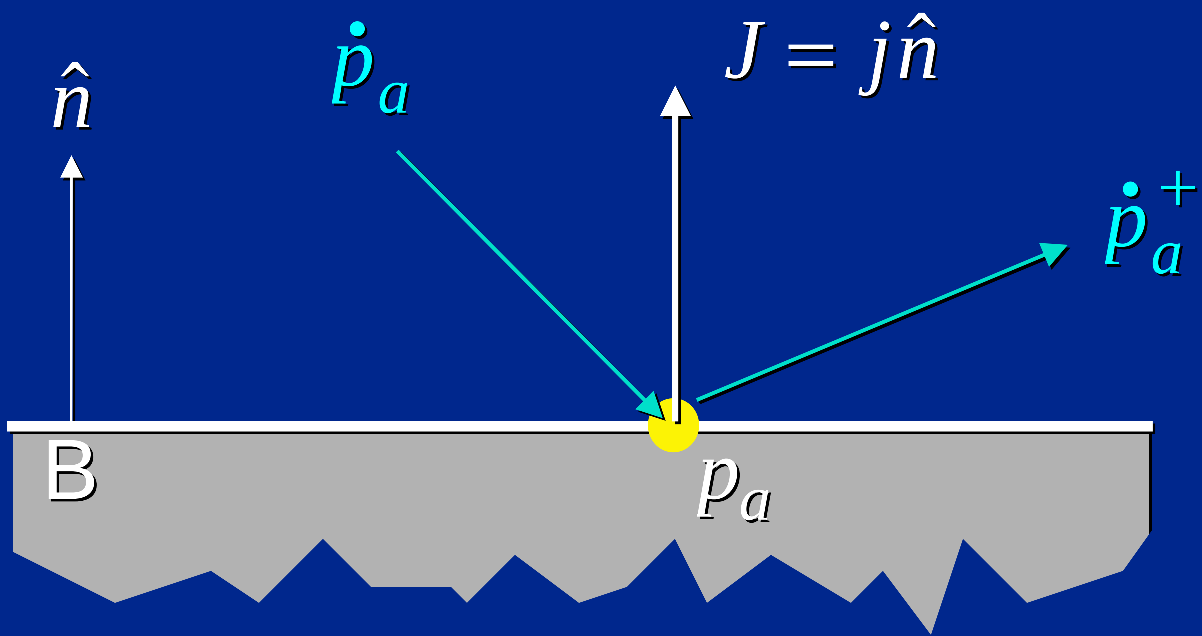
An Infinitely Hard Collision



$$f_{imp} = \infty$$

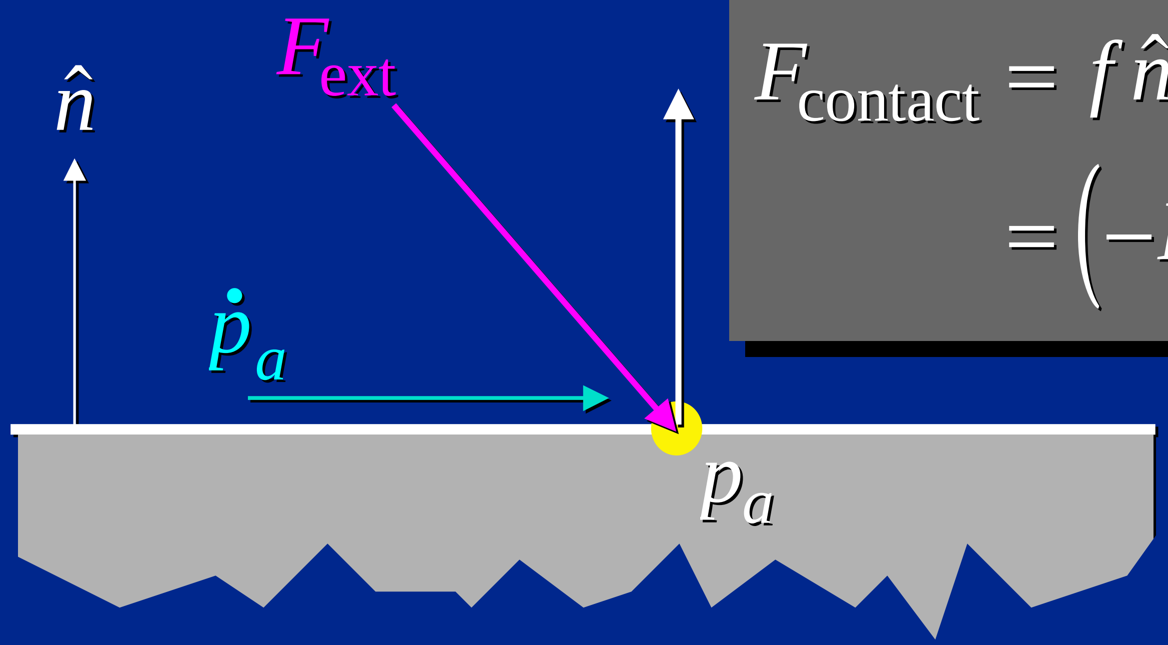
$$\Delta t = 0$$

Colliding Contact



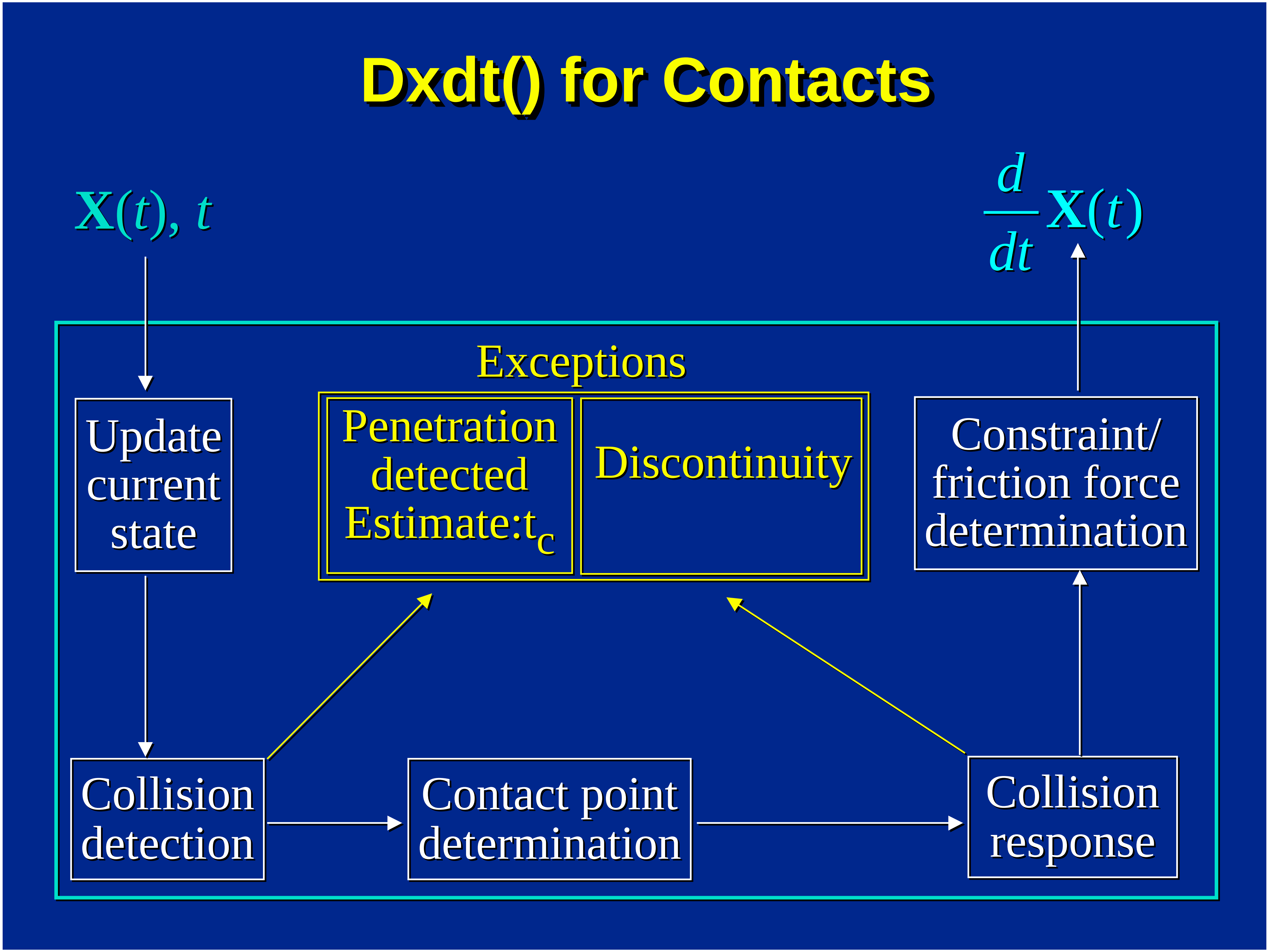
$$\hat{n} \bullet \dot{p}_a < 0 \qquad \dot{p}_a^+ = \frac{J}{m_a} + \dot{p}_a$$

Resting Contact

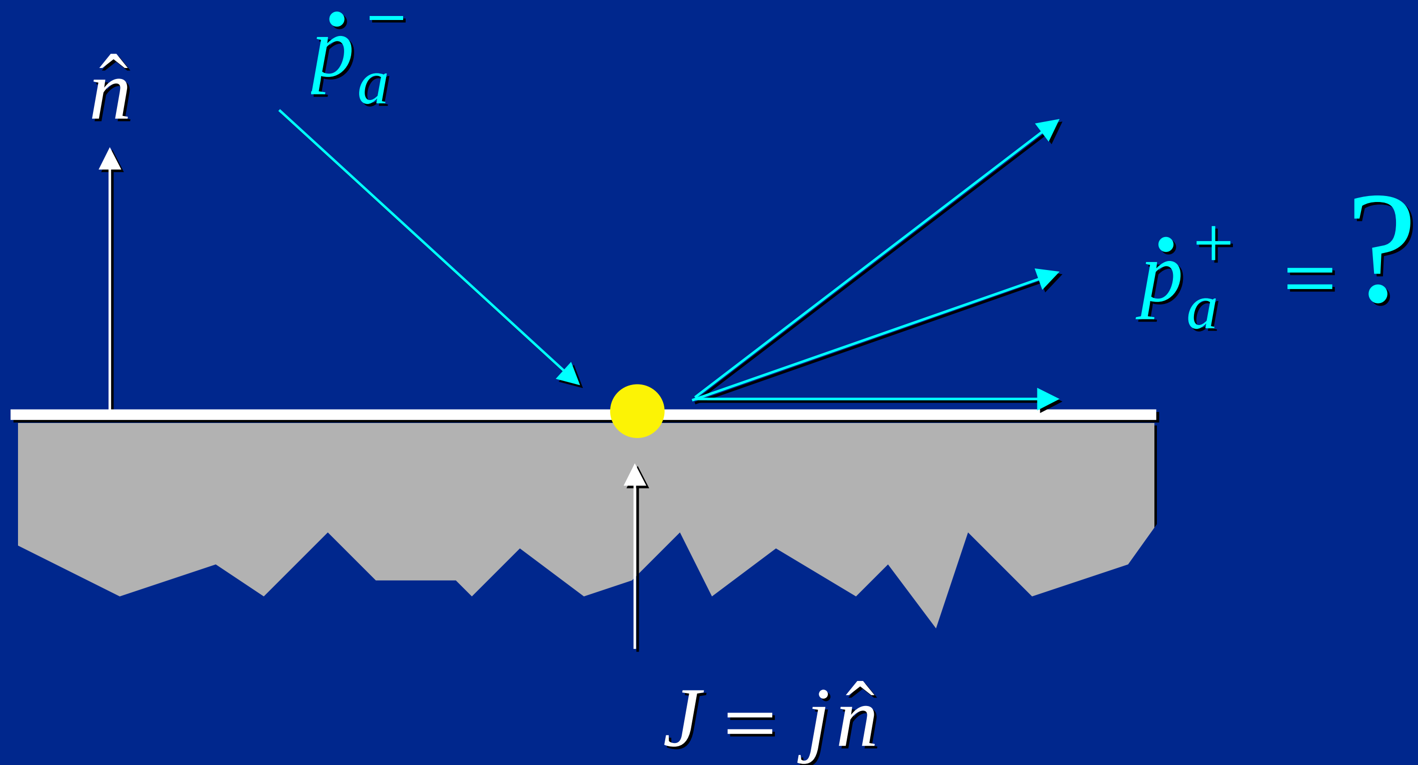


$$F_{\text{contact}} = f\,\hat{n} = \left(-F_{ext} \bullet \hat{n}\right)\hat{n}$$

$$\hat{n} \bullet \dot{p}_a = 0$$

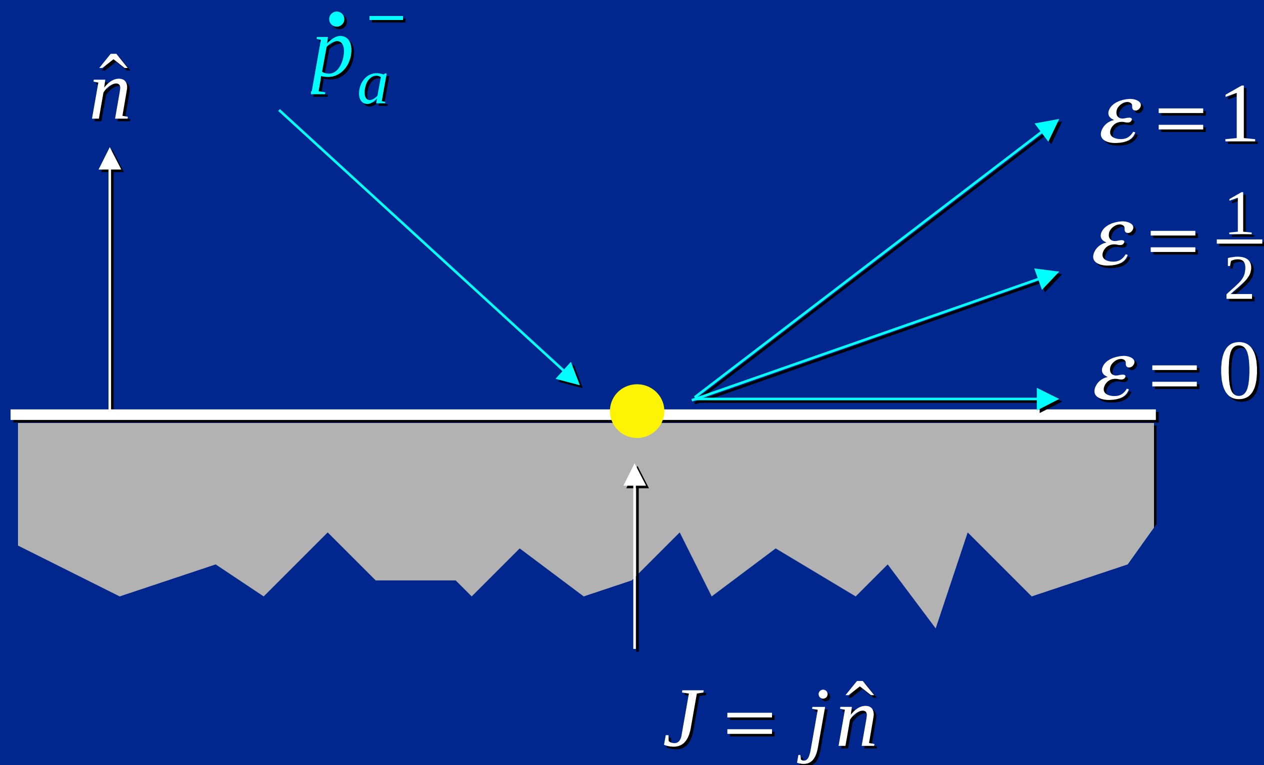

Dxdt() for Contacts
$\mathbf{X}(t), t$
$\frac{d}{dt}\mathbf{X}(t)$
Exceptions
Update current state
Penetration detected Estimate:t_c
Discontinuity
Constraint/ friction force determination
Collision detection
Contact point determination
Collision response

Computing Impulses

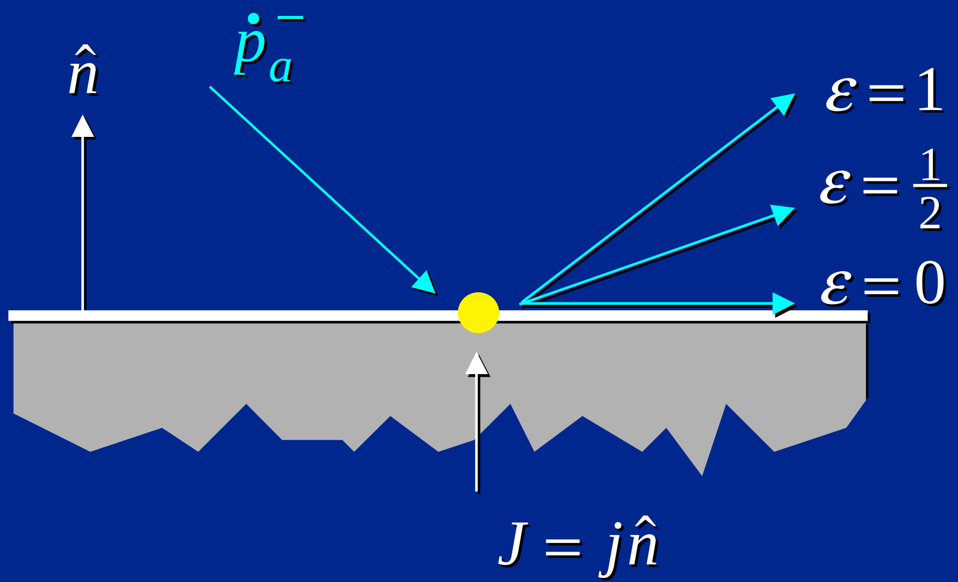


Coefficient of Restitution

$$\hat{n} \bullet \dot{p}_a^+ = -\varepsilon(\hat{n} \bullet \dot{p}_a^-)$$



Computing *j*

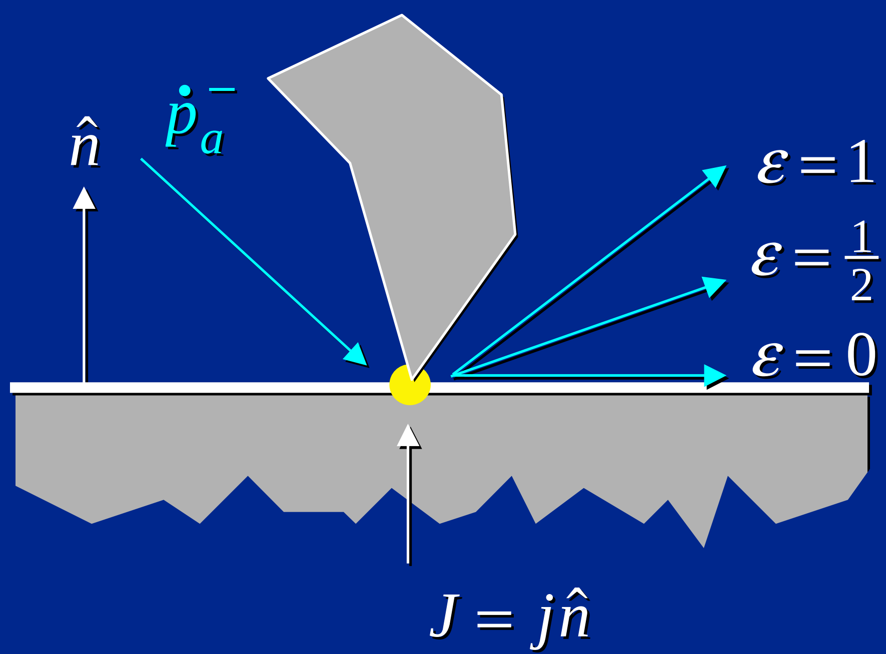


$$\hat{n} \bullet \dot{p}_a^+ = -\varepsilon(\hat{n} \bullet \dot{p}_a^-)$$

$$\dot{p}_a^+ = \frac{j\hat{n}}{m_a} + \dot{p}_a^-$$

$$aj = b$$

Computing *j*



$$\hat{n} \bullet \dot{p}_a^+ = -\varepsilon(\hat{n} \bullet \dot{p}_a^-)$$

$$\dot{p}_a^+ = (m_a, \mathrm{I}_a, v^-, \omega^-, j)$$

$$aj = b$$

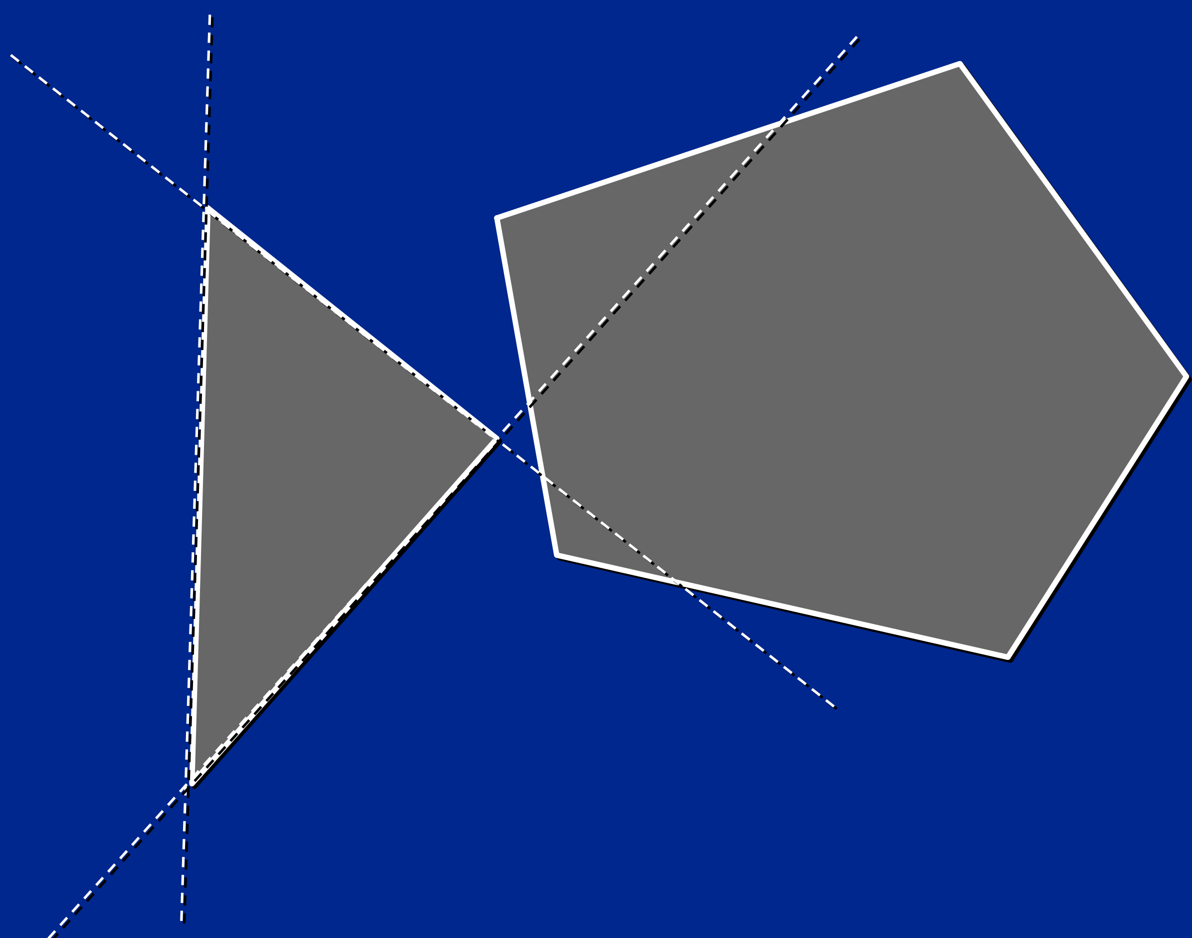
In the Course Notes — Collision Response

Data structures to represent contacts (found by the collision detection phase).

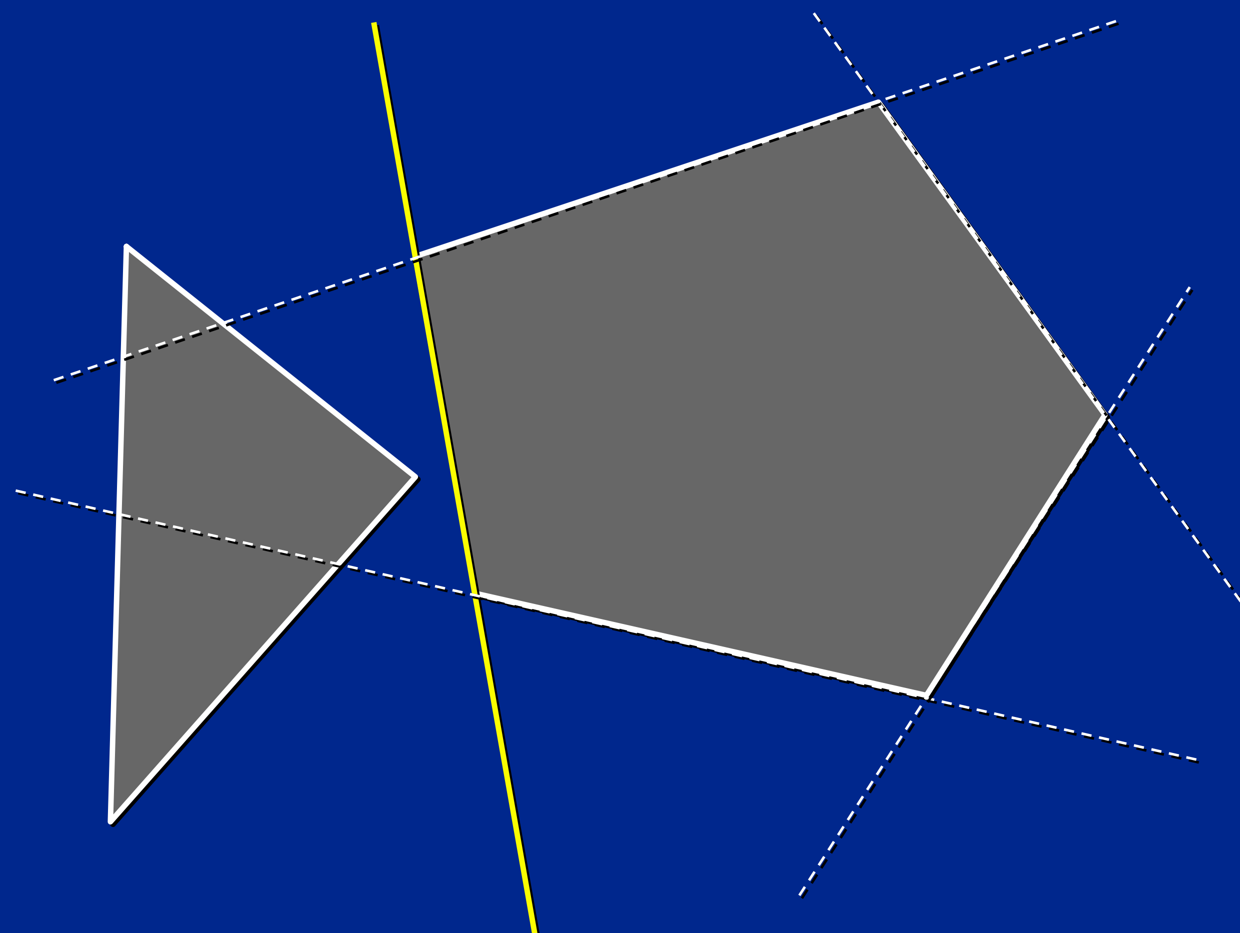
Derivations and code for computing the impulse between two colliding frictionless bodies for a particular coefficient of ε.

Code to detect collisions and apply impulses.

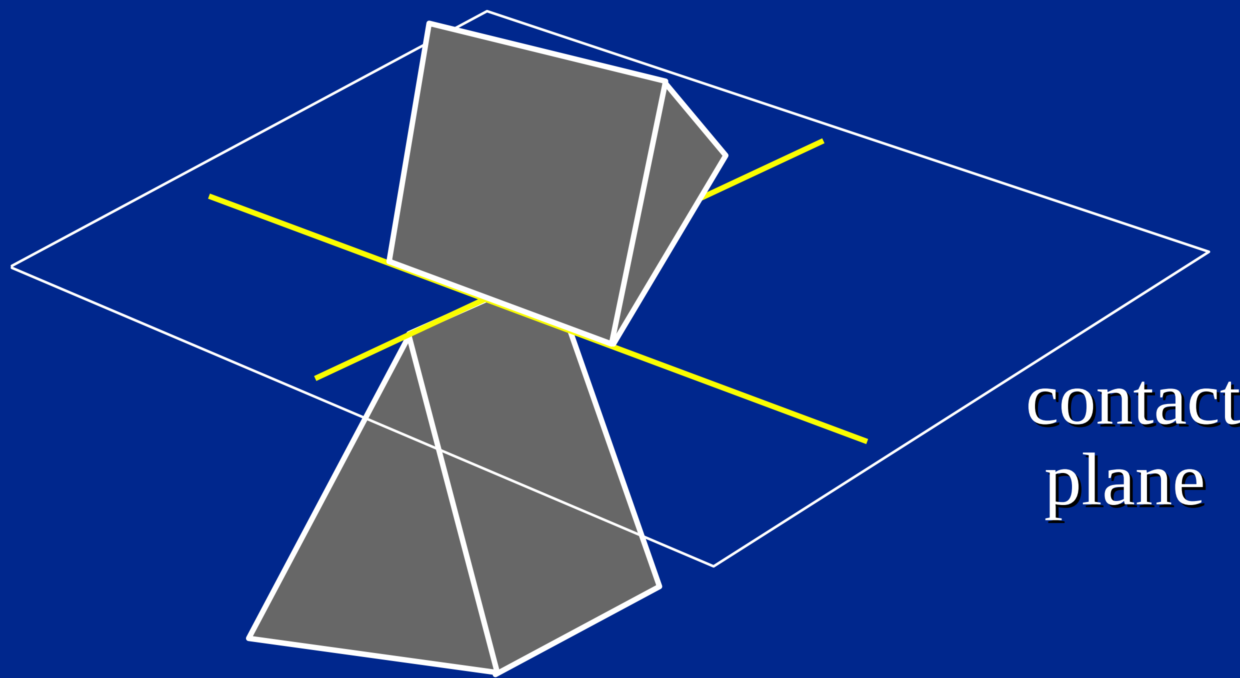
Separating Planes



Separating Planes



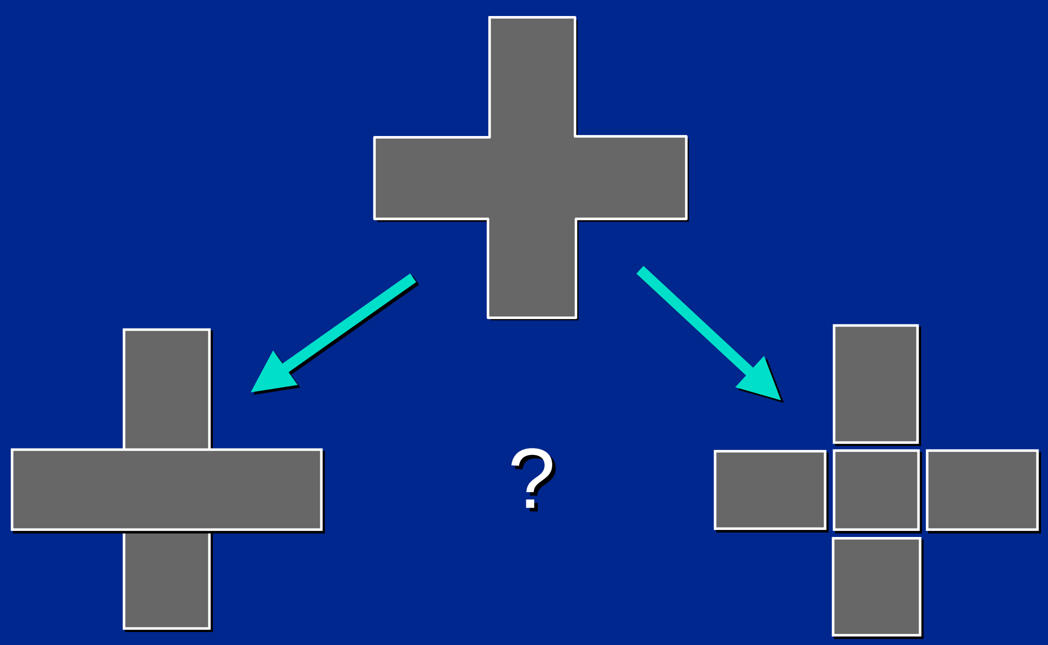
Separating Planes (in 3D)



Does It Really Work?

Yes, but...

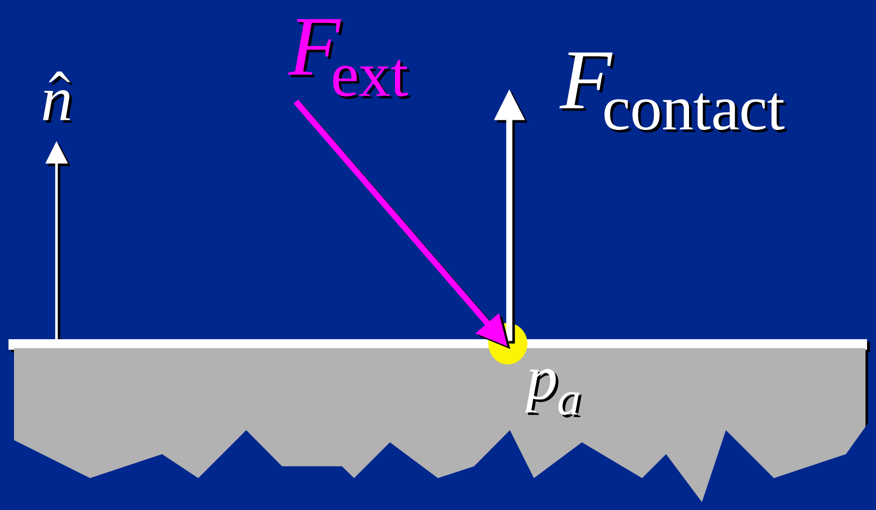
- Requires convex decomposition
- Needs a good decomposition:



An Actual Implementation

- **Coriolis™**—rigid body dynamics engine in Alias|Wavefront's ***Maya*™**
- Fast and reliable
- Compares pairs of polygons from non-convex topologically specified polyhedra (using a coherence-based separating plane approach)
- Hierarchical bounding-box tree to eliminate most false hits

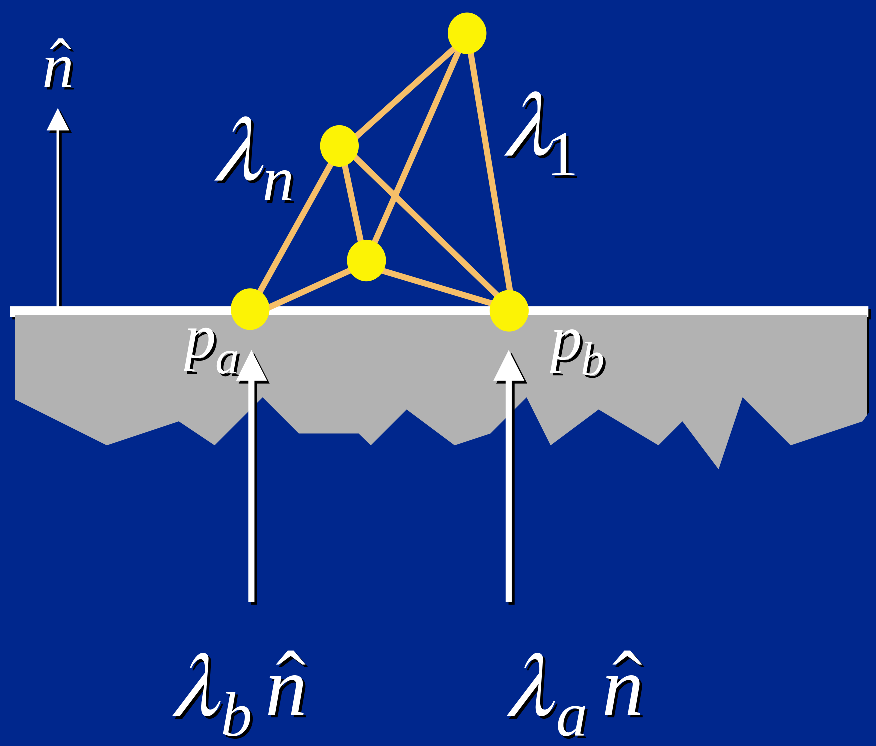
Resting Contact



$$F_{\text{contact}} = f\,\hat{n}$$
$$= \left(-F_{ext} \bullet \hat{n}\right)\hat{n}$$

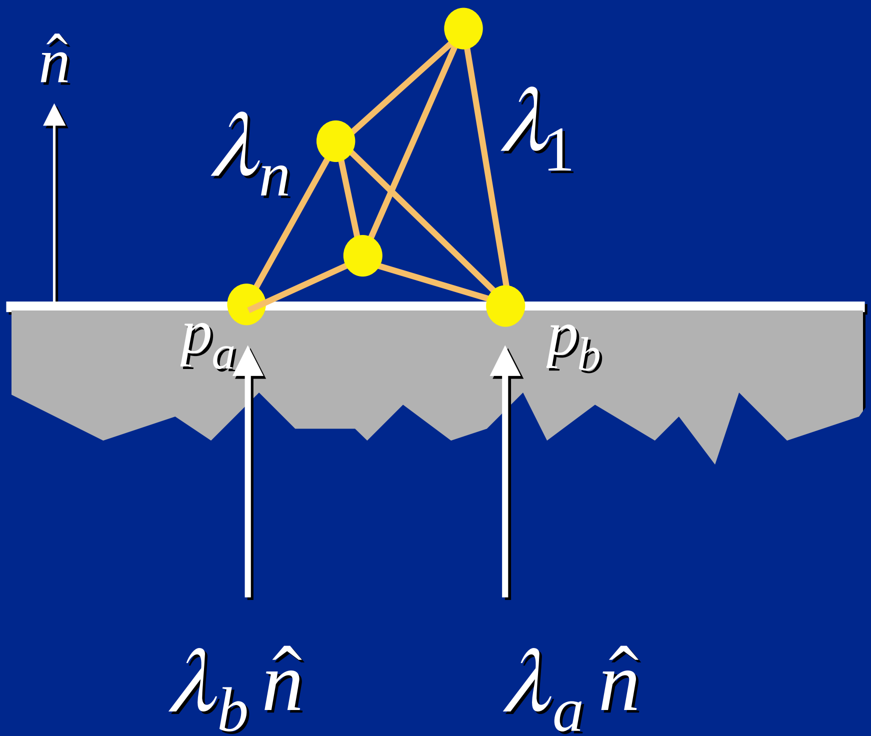
$$f = -F_{ext} \bullet \hat{n}$$
$$(\text{if } f \geq 0)$$

Resting Contact



$$\mathbf{r} = \mathbf{JWJ}^T \begin{pmatrix} \boldsymbol{\lambda} \\ \lambda_a \\ \lambda_b \end{pmatrix} - \mathbf{c} = \mathbf{0}$$

Resting Contact: Quadratic Program



$$\mathbf{r} = \mathbf{JWJ}^T \begin{pmatrix} \boldsymbol{\lambda} \\ \lambda_a \\ \lambda_b \end{pmatrix} - \mathbf{c} \begin{matrix} = \\ \geq \\ \geq \end{matrix} \begin{pmatrix} \mathbf{0} \\ 0 \\ 0 \end{pmatrix}$$

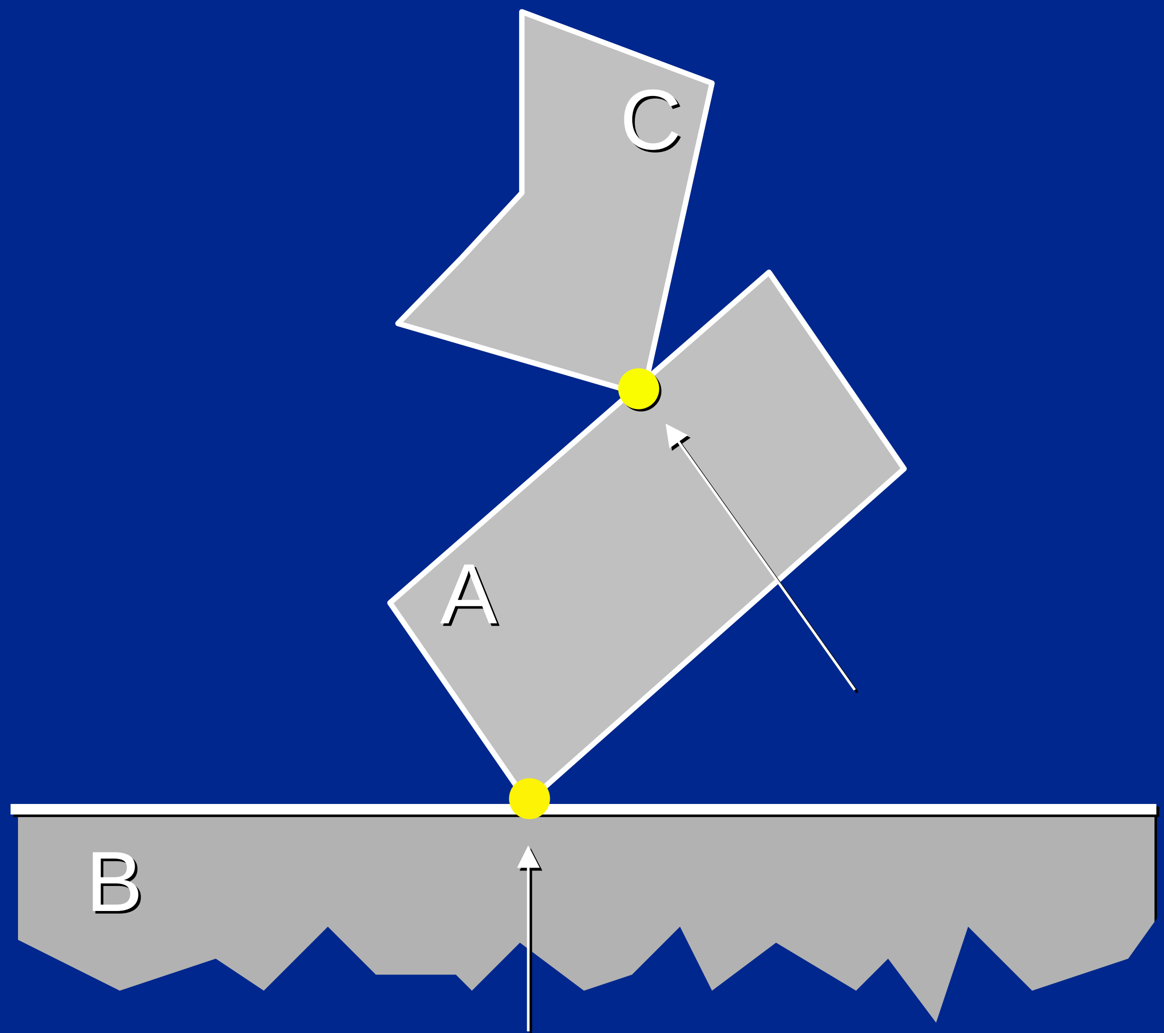
$$\lambda_a \geq 0$$

$$\lambda_b \geq 0$$

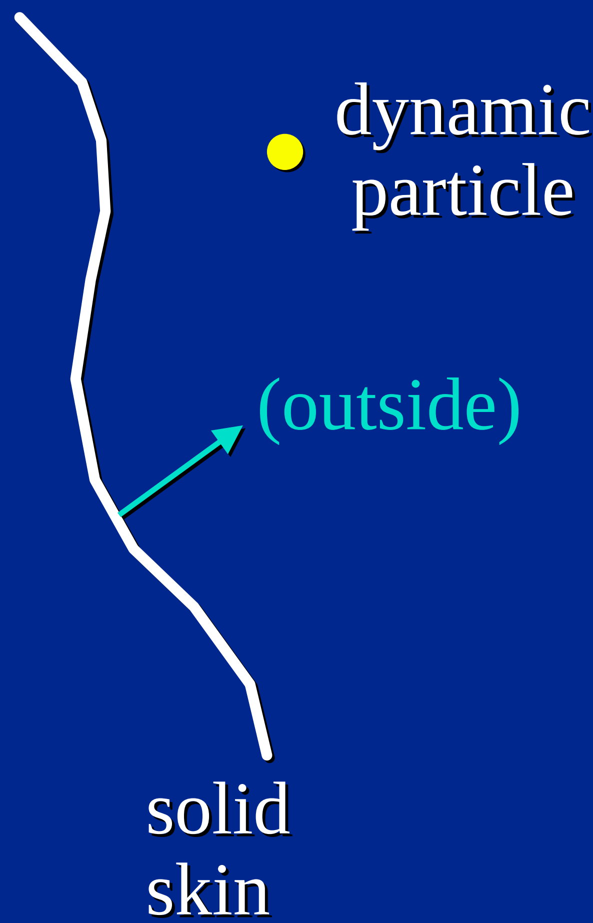
$$\lambda_a = 0 \text{ if } a_{n+1} > 0$$

$$\lambda_b = 0 \text{ if } a_{n+2} > 0$$

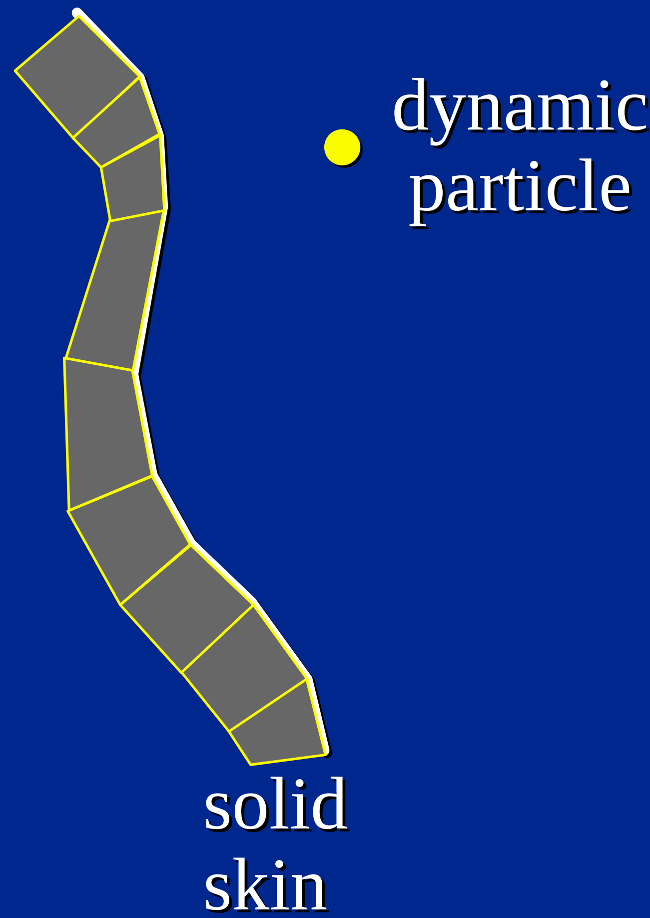
Rigid Bodies: Same Thing!



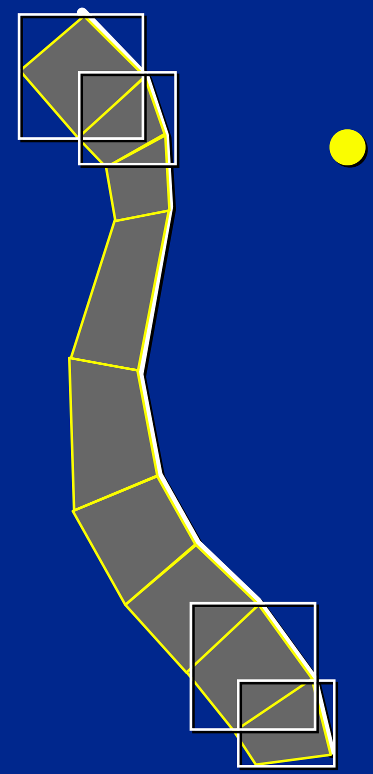
Cloth/Fur Collision Detection



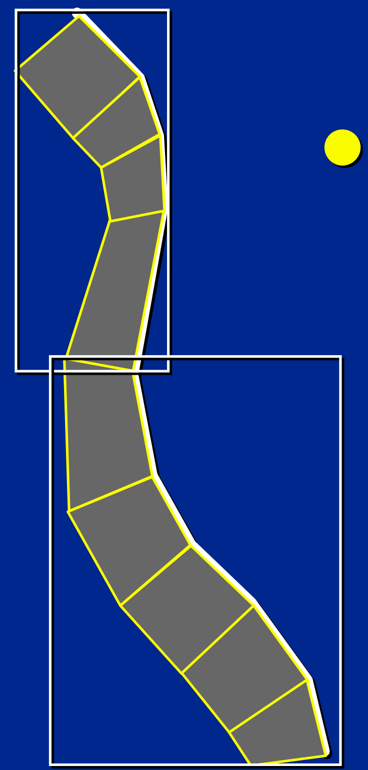
Cloth/Fur Collision Detection



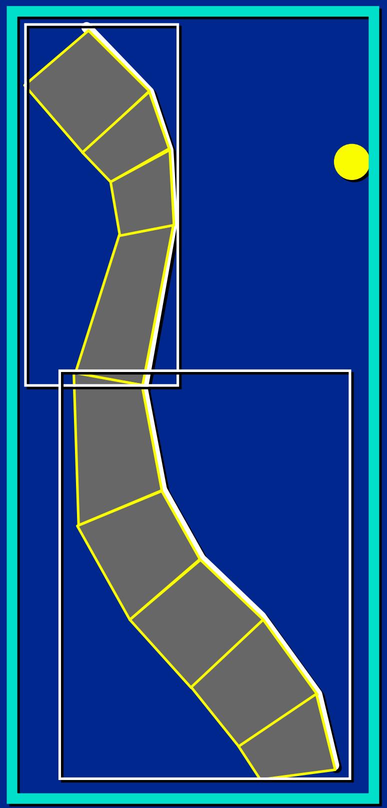
Leaf-level Bounding Boxes



Mid-level Bounding Boxes



Root-level Bounding Box



Cloth/Fur: Establishing Contacts

- New contacts: generally inside
- Too many for partial time steps.
- Gradual correction—bad.
- Arbitrary displacements—worse.
- Solution: combine information about displacements with implicit step method.

Cloth/Fur: Maintaining Constraints

- Penalty force—cloth/cloth contact.
- Exact constraints for cloth/solids:
  - Must work with conjugate-gradient algorithm
  - Lagrange multipliers?
  - Change of variables?